Trial Examination 2006

VCE Further Mathematics Units 3 & 4

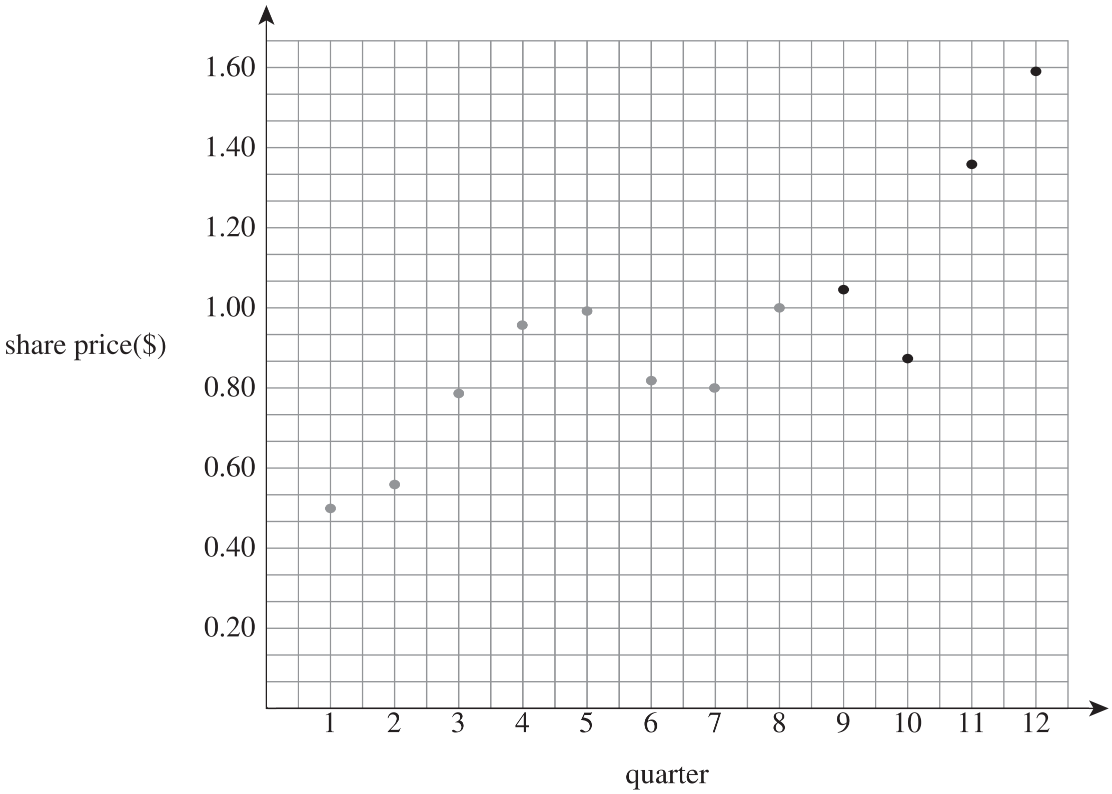
Written Examination 2

Suggested Solutions

SECTION A – DATA ANALYSIS - CORE MATERIAL

Question 1

a.



Answers are the dark points plotted on the graph. A2

2 marks if all correct
1 mark if two are correct
0 marks if one or zero are correct

b. **i.** Using a graphics calculator:

share price = 0.47 + 0.07 × the relevant quarter A2

1 mark for each value

ii. $r = 0.8438$ A1

c. **i.** Use $n = 14$

shareprice = 0.47 + 0.07 × 14 = \$1.45 A1

ii. extrapolation A1

d. **i.** \$0.07 A1

ii. 71% A1

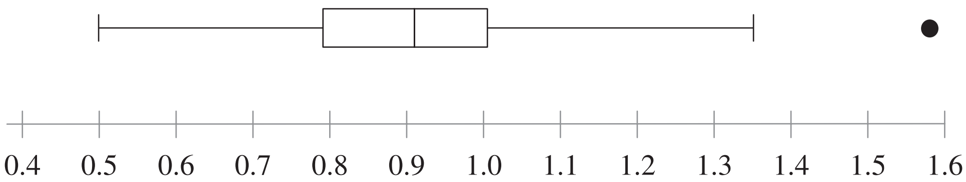
e. **i.** minimum value = 0.49

$Q_1 = 0.79$

median = 0.91

$Q_3 = 1.025$

maximum value = 1.58 M1



A1

ii. We generally refer to any result that is 1.5 × the interquartile range, either to the left of Q_1 or the right of Q_3, as an outlier.

Calculations:
Interquartile range (IQR) = 1.025- 0.79 = 0.235

1.5 × IQR = 0.3525

$Q_1 - 1.5 \times \text{IQR} = 0.79 - 0.3525$

$= 0.4375$

$\therefore$ 0.49 is not an outlier. A1

$Q_3 + 1.5 \times \text{IQR} = 1.025 \times 0.3525$

$= 1.3775$

$\therefore$ \$1.58 is an outlier. A1

f. The pattern of the residual plot seems random and the points are evenly spread above and below the horizontal axis. Therefore it is reasonable to assume that the original data probably has a linear relationship.

A2

1 mark for mentioning a random pattern

1 mark for commenting on the spread of the points above and below the horizontal axis

SECTION B – MODULES

Module 1: Number patterns

Question 1

a. This is clearly an arithmetic sequence. This is a common difference of 8%. A1

b.
$$t_n = a + (n-1)d$$
$$= 20 + (n-1)8$$
$$68 = 20 + 8(n-1)$$
$$48 = 8(n-1)$$
$$6 = n - 1$$
$$n = 7$$

Thus the 68% point would be reached in the seventh month. A1

c. First, find the total of the monthly booking percentages.

$$S_n = \frac{n}{2}[2a + n - 1)d]$$

$$S_6 = \frac{6}{2}[40 + (6-1)8]$$ (*or other correct use of series formula*) M1

$$= 240$$

Now find the revenue.

$$\text{revenue} = \frac{240}{100} \times 5000 = 12\ 000$$

\$12 000 is the predicted mean revenue per room for the first six months. A1

d. This sequence would be geometric. The first term would be 80 and the common ratio would be 0.9.

$$t_n = ar^{n-1}$$
$$32 = 80(0.9)^{n-1}$$
$$0.4 = 0.9^{n-1}$$

either

$$\log_{10}(0.4) = (n-1)\log_{10}(0.9)$$
$$n - 1 = 8.70$$
$$n = 9.7$$

OR by trial and error

$$t_9 = 34.4$$
$$t_{10} = 31.0$$ M1

Thus month 10 would be the closest month. A1

e. This is an infinite series.

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{80}{1-0.9}$$

$$= 800$$ M1

Thus each room would be expected to lose 800% of its monthly revenue.
This is \$40 000 per room. A1

Question 2

a. The balance is reduced by 2000 after the interest is added. It is important that interest is not added before the 2000 is subtracted.

$t_{n+1} = 1.01t_n - 2000;\ t_0 = 100\ 000$ M1 A1

1 mark for correct format, even if initial condition is omitted or the subscripts are incorrect
an additional 1 mark for giving a completely correct answer

b. $t_{n+1} = 1.008(t_n - 1000);\ t_0 = 100\ 000$

$t_1 = 1.008(100\ 000 - 1000)$

$= 99\ 792$ M1

$t_2 = 1.008(99\ 792 - 1000)$

$= 99\ 582.34$

Thus the balance after two months is \$99 582.32 A1

c. From pulling the equation apart, it becomes clear that it is applicable to a loan of \$100 000 with interest at 0.8% per month and repayments of \$1000 per month before interest is applied. A1

d. We need an equation similar to that in parts **b** and **c**, except that

- the repayment of \$1000 must be replaced by one of 950;
- an additional amount, related to the term two preceding the current one, must also be subtracted.

M1

1 mark for demonstrating an understanding of the process

$t_{n+1} = 1.008(t_n - 950 - 0.03t_{n-1})$ A1

(All students who manage to correctly obtain the difference equation should be awarded the method mark in addition to the answer mark regardless of whether a written explanation has been provided.)

Module 2: Geometry and trigonometry

Question 1

a. $90° - 42° = 48°$ A1

b. $a^2 = b^2 + c^2 - 2bc\cos(A)$

$a^2 = 1.4^2 + 1.24^2 - 2 \times 1.4 \times 1.24 \times \cos(48°)$

$a = 1.0836\ldots$ $\quad$ $a \cong 1.08$ km A1

c. $\text{area} = \frac{1}{2}bc\sin(A)$

$\text{area} = \frac{1}{2} \times 1400 \times 1240 \times \sin(48°)$

area = 645 049.70

area $\cong$ 645 000 m^2 A1

d. $\frac{\text{adjacent}}{\text{hypotenuse}} = \cos(42°)$

$\frac{\text{adjacent}}{1400} = \cos(42°)$

adjacent = 1040.40…

adjacent = 1040 m A1

e. $a^2 = b^2 + c^2 - 2bc\cos(A)$

$a^2 = 1.55^2 + 1.4^2 - 2 \times 1.55 \times 1.4 \times \cos(48°)$

$a = 1.4584...$

$a \approx 1.21$ km A1

f. $\underline{\angle ABC}$

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(B) = \frac{1.24^2 + 1.0836^2 - 1.4^2}{2 \times 1.24 \times 1.0836}$$

$$\cos(B) = 0.279753...$$

$$B = 73.7545°...$$ A1

$\underline{\angle ABB'}$

$$\angle ABB' = 180° - 73.7545° = 106.24548°$$ A1

$\underline{\text{area } ABB'}$

$$\text{area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 1083.6874 \times 310 \times \sin(106.24548°)$$

$$= 161\ 264.7668\ \text{km}^2$$

$$\approx 161\ 300\ \text{m}^2 \text{ (to nearest 100 m}^2\text{)}$$ A1

Question 2

a. $470 - 455 = 15$ m — A1

b.

$$\frac{\text{opposite}}{\text{adjacent}} = \tan(3°11')$$

$$\frac{15}{\text{adjacent}} = \tan(3°11')$$

$$\frac{15}{\tan(3°11')} = \text{adjacent}$$

$$269.7022\ldots = \text{adjacent}$$

$$270 \text{ m} = \text{adjacent}$$ A1

Question 3

a. Circle (or end area):

$A = \pi r^2$

$A = \pi \times 7^2$

$A = 153.93804 \text{ cm}^2$ — A1

$V = \text{end area} \times \text{height}$

$V = 153.93804 \times 200$

$V = 30\ 787.608\ldots$

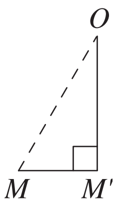
$V \cong 30\ 788 \text{ cm}^3$ — A1

b. k = ratio of corresponding 'distances'
then k^3 = ratio of corresponding 'volumes'

$k = \frac{280}{140} = 2$

$\Rightarrow k^3 = 2^3 = 8$ — A1

c.



Angle $OMM' = 60°$ — M1

$$\text{Area } OMM' = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 6.25 \times 12.5 \times \sin(60°)$$

$$= 33.83 \text{ m}^2$$

$$\text{Area of lawn} = 12 \times \text{area } OMM'$$

$$= 12 \times 33.83$$

$$= 405.95$$

$$= 406 \text{ m}^2$$ A1

Module 3: Graphs and relations

Question 1

a. $6x + 8y \leq 7200 \quad \ldots(1)$

$45x + 50y \leq 50\ 000$

$\therefore\ 9x + 10y \leq 10\ 000 \quad \ldots(2)$

$2x + 5y \leq 4000 \quad \ldots(3)$

$x \geq 0$

$y \geq 0$

A2

1 mark each for any two of the equations (1), (2) and (3))

b. $P = 12x + 16y$ A1

c. $6x + 8y \leq 7200$

x-intercept:

$y = 0$

$6x = 7200$

$x = 1200$

$(1200, 0)$

y-intercept:

$8y = 7200$

$y = 900$

$(0, 900)$

$9x + 10y \leq 10\ 000$

x-intercept:

$y = 0$

$9x = 10\ 000$

$x = 1111$

$(1111, 0)$

y-intercept:

$10y = 10\ 000$

$y = 1000$

$(0, 1000)$

$2x + 5y \leq 4000$

x-intercept:

$y = 0$

$2x = 4000$

$x = 2000$

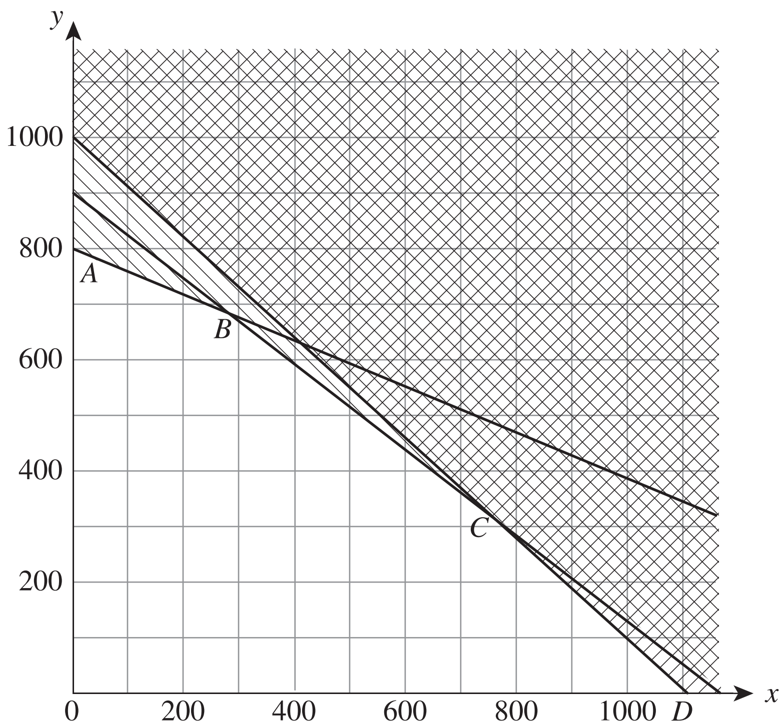
y-intercept:

$x = 0$

$5y = 4000$

$y = 800$

Shows correct method to draw lines. M1



The required region in the above graph is the unshaded section.

Correct line locations. A1

Correct shading of regions. A1

d. It will be necessary to determine all of the intersections on the boundary of the required region shown. We already have some of these as they are intercepts with the axes.
A (0, 800); *D* (1111, 0)

The remainder must be determined by finding the intersections of the lines.
Point *B*:

$6x + 8y = 7200 \quad \ldots(1)$

$2x + 5y = 4000 \quad \ldots(3)$

$(3) \times 3$:

$6x + 15y = 12\ 000 \quad \ldots(3a)$

$(3a) - (1)$:

$7y = 4800$

$y = 685.7$

$2x + 5(685.7) = 4000$

$2x + 3428.5 = 4000$

$2x = 571.5$

$x = 285.7$

B (285.7, 685.7)
Shows correct method to find intersections. M1

Point *C*:

$6x + 8y = 7200 \quad \ldots(1)$

$9x + 10y = 10\ 000 \quad \ldots(2)$

$(1) \times 1.5$:

$9x + 12y = 10\ 800 \quad \ldots(1a)$

$(1a) - (2)$:
Correct choice of lines to find the intersection between. M1

$2y = 800$

$y = 400$

$6x + 8(400) = 7200$

$6x = 4000$

$x = 666.7$

C (666.7, 400) A1

e. It is only necessary to determine the values of the profit function at all points *A* to *D*.

	coordinates	profit
A	(0, 800)	12 800
B	(285.7, 685.7)	14 399.6
C	(666.7, 400)	14 400.4
D	(1111, 0)	13 333

M1

Of these points, *C* is marginally the best. Thus a maximum profit of $14 400.40 can be made by a mean of 666.7 standard pairs and 400 deluxe pairs being made each week. A1

Question 2

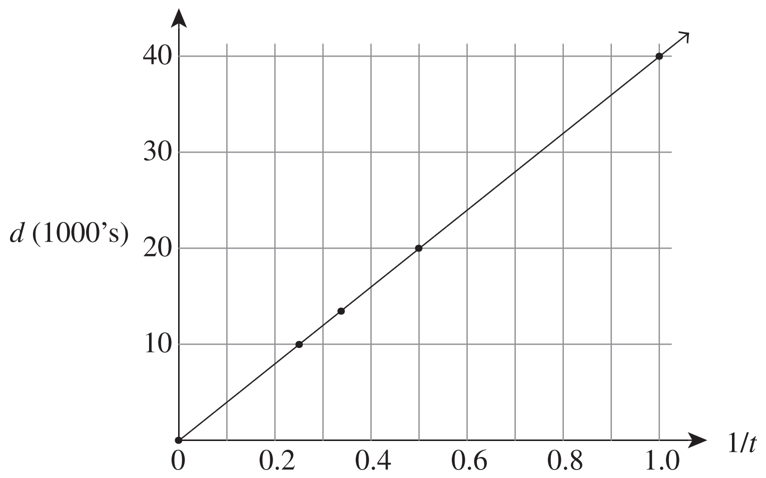
a.

week (t)	1	2	3	4
production (p)	10 000	30 000	36 667	40 000
deficit (d)	40 000	20 000	13 333	10 000

A1

b. To be a quadratic relationship, each of the values of d must be proportional to t^2. This is untrue since the first is $40\,000 \times t^2$ while the second is $5000 \times t^2$. A1

c. The horizontal axis should be labelled $\frac{1}{t}$ and the graph should be:



M1

The gradient is 40 000 and thus k is 40 000. A1

Module 4: Business-related mathematics

Question 1

a. 10% of \$3495 $= 0.1 \times 3495$

$= 349.5$

Jason will need to withdraw \$350. A1

b. amount borrowed $= 0.9 \times 3495$

$= 3145.5$

interest $= \dfrac{Prt}{100}$

$= 3145.5 \times 0.08 \times 3$

$= 754.92$

The total interest is \$755. A1

repayment $= \dfrac{(3145.5 + 754.92)}{(3 \times 26)}$

$= 50.01$

The fortnightly repayment is \$50.00. A1

c. total cost $= 3495 + 754.90$

$= 4249.90$

The total cost is \$4250. A1

d. effective rate $= \dfrac{r \times 2n}{(n+1)}$

$= 8 \times 2 \times \dfrac{78}{79}$

$= 15.8$

The effective interest rate is 15.8%. A1

Question 2

a. 5% of 52 000 + 9% × 0.85 × 52 000 = 6578 M2

monthly payment $= \dfrac{6578}{12}$

$= 548.17$ A1

The monthly payment is \$548.

b. Use the TVM Solver.

```
N=384
I%=6.5
PV=0
PMT=-548
FV=?
P/Y=12
C/Y=12
PMT:END BEGIN
```

(for values) M1

The FV would be 704 103.97.

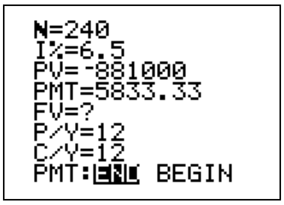
The amount in the fund would be \$704 100. A1

c. future salary $= 52\ 000 \times 1.02^{32}$

$= 97\ 996.11$

Her future salary would be \$98 000. A1

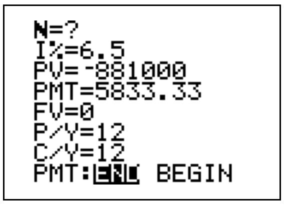
d. **i.** Use the TVM Solver.



(*for values*) M1

The FV would be 360 542.
Mary would have \$360 500. A1

ii. Use a TVM solver.



(*for values*) M1

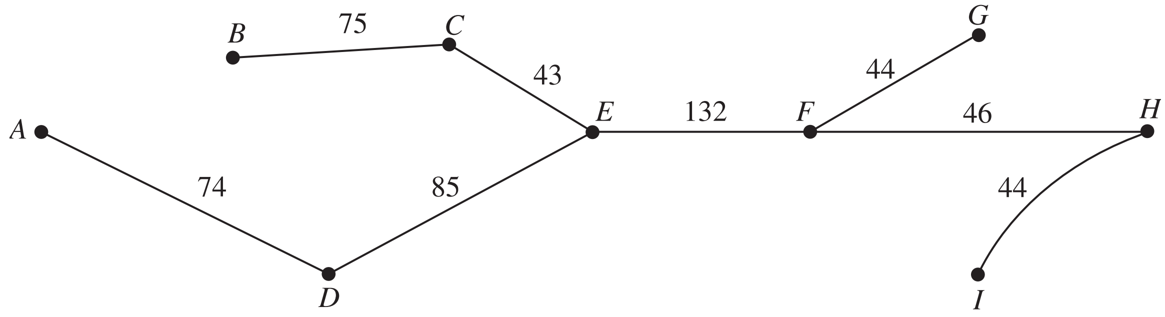
The value of N is 315.46
The funds would last for 26.3 years. A1

Module 5: Networks and decision mathematics

Question 1

a. $85 + 132 + 44 = 261$ km A1

b.



A1

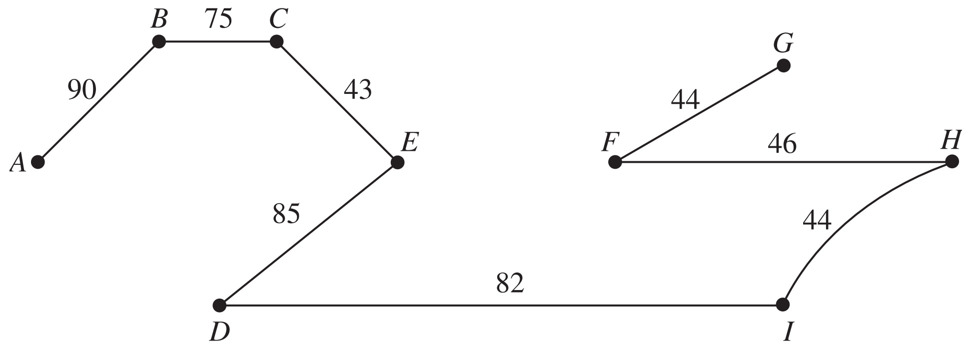
$74 + 75 + 43 + 85 + 132 + 44 + 46 + 44 = 543$ km A1

c. previously: $D \to E \to F \to G = 261$ km

new: $D \to I \to H \to G = 82 + 44 + 54 = 180$ km

saving $= 261 - 180 = 81$ km A1

d.



A1

e. $90 + 75 + 43 + 85 + 82 + 44 + 46 + 44 = 509$ km A1

f. No it is not possible. A1
Two vertices have an odd degree (vertex D and vertex E). Therefore it is not possible to travel along each edge only once and start and finish at the same port. A1

g. Vertex D and vertex E. These vertices are the only vertices with an odd degree. A1

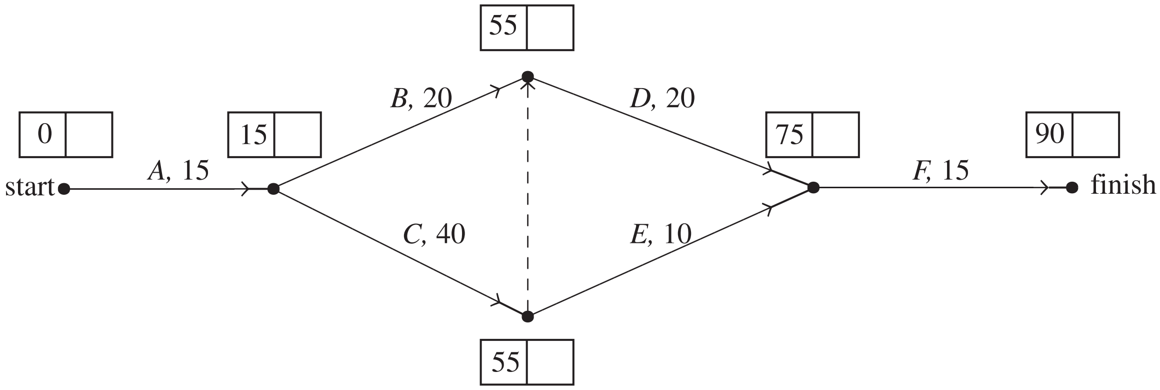
h. Euler circuit A1

Question 2

a. 55 minutes A1

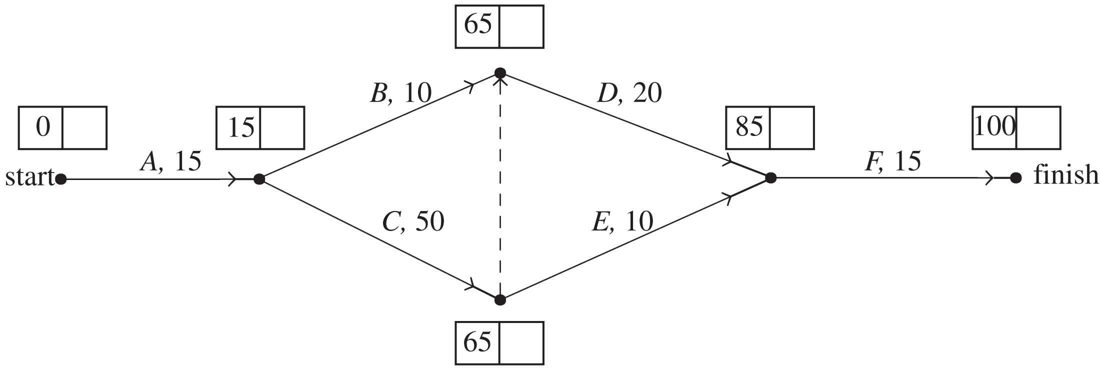
b. 90 minutes A1

c.



90 minutes A1

d.



100 minutes A1

e. Activity *B* should be delayed. A1

Module 6: Matrices

Question 1

a. $$\begin{bmatrix} C \\ S \\ A \\ B \end{bmatrix} = \begin{bmatrix} 0 & 10 & 3 & 7 \\ 10 & 1 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 1 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} E \\ I \\ H \\ R \end{bmatrix}$$

A1

b. $$\begin{bmatrix} C \\ S \\ A \\ B \end{bmatrix} = \begin{bmatrix} 0 & 10 & 3 & 7 \\ 10 & 1 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 1 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 120 \\ 40 \\ 210 \\ 50 \end{bmatrix}$$

M1

$$= \begin{bmatrix} 1380 \\ 2070 \\ 680 \\ 860 \end{bmatrix}$$

A1

c. $$\begin{bmatrix} 45 \\ 64 \\ 21 \\ 26 \end{bmatrix} = \begin{bmatrix} 0 & 10 & 3 & 7 \\ 10 & 1 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 1 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} E \\ I \\ H \\ R \end{bmatrix}$$

Using the matrix inverse from the graphics calculator:

$$\begin{bmatrix} 0.05 & 0.1 & 0.15 & 0.3 \\ 0.23 & 0.06 & 0.11 & 0.38 \\ 0.01 & 0.22 & 1.57 & 0.94 \\ 0.19 & 0.18 & 0.83 & 0.14 \end{bmatrix} \begin{bmatrix} 45 \\ 64 \\ 21 \\ 26 \end{bmatrix} = \begin{bmatrix} E \\ I \\ H \\ R \end{bmatrix}$$

M1

$$\begin{bmatrix} 4 \\ 2 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} G \\ H \\ D \\ S \end{bmatrix}$$

A1

Question 2

a. Envirosafe: $0.62 \times 21 = 13.02\%$

Industrial: $0.62 \times 20 = 12.4\%$

Home: $0.62 \times 46 = 28.52\%$

Slow-release: $0.62 \times 13 = 8.06\%$

A1

b. $N_0 = \begin{bmatrix} 0.1302 \\ 0.124 \\ 0.2852 \\ 0.0806 \\ 0.38 \end{bmatrix}$

A1

c. $T = \begin{bmatrix} 0.80 & 0.01 & 0.02 & 0.04 & 0.07 \\ 0.01 & 0.67 & 0.10 & 0.06 & 0.09 \\ 0.02 & 0.14 & 0.66 & 0.10 & 0.07 \\ 0.09 & 0.07 & 0.10 & 0.73 & 0.04 \\ 0.08 & 0.11 & 0.12 & 0.07 & 0.73 \end{bmatrix}$

A1

d. $T = \begin{bmatrix} 0.80 & 0.01 & 0.02 & 0.04 & 0.07 \\ 0.01 & 0.67 & 0.10 & 0.06 & 0.09 \\ 0.02 & 0.14 & 0.66 & 0.10 & 0.07 \\ 0.09 & 0.07 & 0.10 & 0.73 & 0.04 \\ 0.08 & 0.11 & 0.12 & 0.07 & 0.73 \end{bmatrix} \begin{bmatrix} 0.1302 \\ 0.124 \\ 0.2852 \\ 0.0806 \\ 0.38 \end{bmatrix}$

$= \begin{bmatrix} 0.1409 \\ 0.1519 \\ 0.2429 \\ 0.1230 \\ 0.3413 \end{bmatrix}$

Clearly all of the types of cleaner produced by Bill, except the **Home** type, have gained, but the total other brands have lost market share. A1

e. $T^2 N_0 = \begin{bmatrix} 0.1479 \\ 0.1656 \\ 0.2206 \\ 0.1510 \\ 0.3149 \end{bmatrix}$

These are the numbers of each type for 2008. This represents a continuation of the changes that occurred in 2007. A1

$T^3 N_0 = \begin{bmatrix} 0.1525 \\ 0.1719 \\ 0.2089 \\ 0.1698 \\ 0.2970 \end{bmatrix}$

These are the proportions of total market for each of the types of cleaner for 2009.
Again, the trends evident earlier are continuing. A1

f. This question will require the inverse of T.

$$T = \begin{bmatrix} 1.2685 & 0.0085 & -0.0094 & -0.0575 & -0.1186 \\ 0.0121 & 1.5701 & -0.1940 & -0.0867 & -0.1714 \\ -0.0050 & -0.2991 & 1.6081 & -0.1852 & -0.1067 \\ -0.1500 & -0.1008 & -0.1887 & 1.4153 & -0.0326 \\ -0.1256 & -0.1787 & -0.2160 & -0.0859 & 1.4294 \end{bmatrix}$$

M1

$$T^{-1}N = \begin{bmatrix} 0.1138 \\ 0.0688 \\ 0.3654 \\ 0.0158 \\ 0.4361 \end{bmatrix}$$

These results are possible. It is likely, however, that problems will occur as attempts are made to determine the proportions for preceding years. The year 2004 result would be:

$$(T^{-1})^2 N = \begin{bmatrix} 0.0889 \\ -0.0375 \\ 0.5170 \\ -0.0848 \\ 0.5165 \end{bmatrix}$$

Negative proportions are impossible and so it is clear that this method does have problems. Results are not realistic. A1

g. In order to determine whether a stable state exists, it is necessary to determine the behaviour of the matrix T when applied a large number of times. Try T^{64} (64th transition):

$$T^{64} = \begin{bmatrix} 0.1612 & 0.1612 & 0.1612 & 0.1612 & 0.1612 \\ 0.1733 & 0.1733 & 0.1733 & 0.1733 & 0.1733 \\ 0.1960 & 0.1960 & 0.1960 & 0.1960 & 0.1960 \\ 0.2097 & 0.2097 & 0.2097 & 0.2097 & 0.2097 \\ 0.2598 & 0.2598 & 0.2598 & 0.2598 & 0.2598 \end{bmatrix}$$

Clearly this matrix is stable. Every row is identical (to four decimal places). M1

$$T^{40} N_0 = \begin{bmatrix} 0.1612 \\ 0.1733 \\ 0.1960 \\ 0.2097 \\ 0.2598 \end{bmatrix}$$

Thus the Envirosafe market share is 16%, Industrial is 17%, Home is 20% and Slow-release is 21%. Therefore 26% of sales are for other brands. A1